```
SetDirectory@NotebookDirectory[];

ManToGif[man_, name_String, step_Integer, durations_: 0.6] :=
 Export[name <> ".gif", Import[Export[name <> Which[$OperatingSystem == "MacOSX",
      ".mov", $OperatingSystem == "Windows", ".avi"], man], "ImageList"][[
   1 ;; -1 ;; step]], "DisplayDurations" → durations]

fontsize = 18;
(* http://mathematica.stackexchange.com/questions/18012/label-area-on-plot *)
braceLabel[{p1_, p2_}, lbl_, scale_: .02] :=
  {Arrowheads[{{scale, 0, {Graphics@Circle[{1, -1}, 1, {Pi / 2, Pi}], -1}}, {scale, 1,
       {Graphics[{Circle[{-1, 1}, 1, {-Pi / 2, 0}], Inset[lbl, {0, 2}]}], 1}}}],
    Arrow[{p1, (p1 + p2) / 2}], Arrowheads[
     {{scale, 0, {Graphics@Circle[{1, 1}, 1, {Pi, 3 Pi / 2}], -1}}, {scale, 1,
       {Graphics@Circle[{-1, -1}, 1, {0, Pi / 2}], 1}}}], Arrow[{(p1 + p2) / 2, p2}]};
```

```mathematica
man = Manipulate[
  Show[
   Plot[{Sin[x], a Sin[x]}, {x, -2 π, 4 π},
    Ticks →
     {{-2 π, -(3 π)/2, -π, -π/2, π/2, π, (3 π)/2, 2 π, (5 π)/2, 3 π, (7 π)/2, 4 π}, {-3, -2, -1, 1, 2, 3}},
    PlotRange → {{-2 π, 4 π + 2}, {-4.2, 3.8}},
    PlotStyle → {Directive[Red, AbsoluteThickness[2]],
      Directive[Blue, AbsoluteThickness[2]]},
    PlotLegends → LineLegend[{"  f(x) =    sin(x)",
       "a f(x) = a sin(x)\n          = " <> ToString[a] <> " sin(x)"},
      LegendFunction → "Frame", LegendMargins → 6, LabelStyle →
       Directive[FontFamily → "Courier"]], BaseStyle → {FontSize → fontsize}],
   Graphics[If[a ≠ 0, braceLabel[{{4 π + 0.4, Abs[a]}, {4 π + 0.4 , 0}},
       "amplitude", If[Abs[a] < 1.5, 0.02/1.5 Abs[a], 0.02]], {}]],
   Graphics[If[a ≠ 0, braceLabel[{{2 π , -3 - 0.4}, {0, -3 - 0.4}},
       Rotate["period", 180 Degree], 0.02], {}]], ImageSize → 800
   ]
  , {{a, 1, "amplitude, a = "}, -3, 3},
  BaseStyle → {FontSize → fontsize}, Initialization :→ (fontsize = 18;
    (* http://mathematica.stackexchange.com/questions/18012/label-
       area-on-plot *)
    braceLabel[{p1_, p2_}, lbl_, scale_: .02] :=
     {Arrowheads[{{scale, 0, {Graphics@Circle[{1, -1}, 1, {Pi / 2, Pi}], -1}},
        {scale, 1, {Graphics[{Circle[{-1, 1}, 1, {-Pi / 2, 0}], Inset[lbl, {0, 2}]}],
          1}}}], Arrow[{p1, (p1 + p2) / 2}],
      Arrowheads[{{scale, 0, {Graphics@Circle[{1, 1}, 1, {Pi, 3 Pi / 2}], -1}},
        {scale, 1, {Graphics@Circle[{-1, -1}, 1, {0, Pi / 2}], 1}}}],
      Arrow[{ (p1 + p2) / 2, p2}]};)]
```
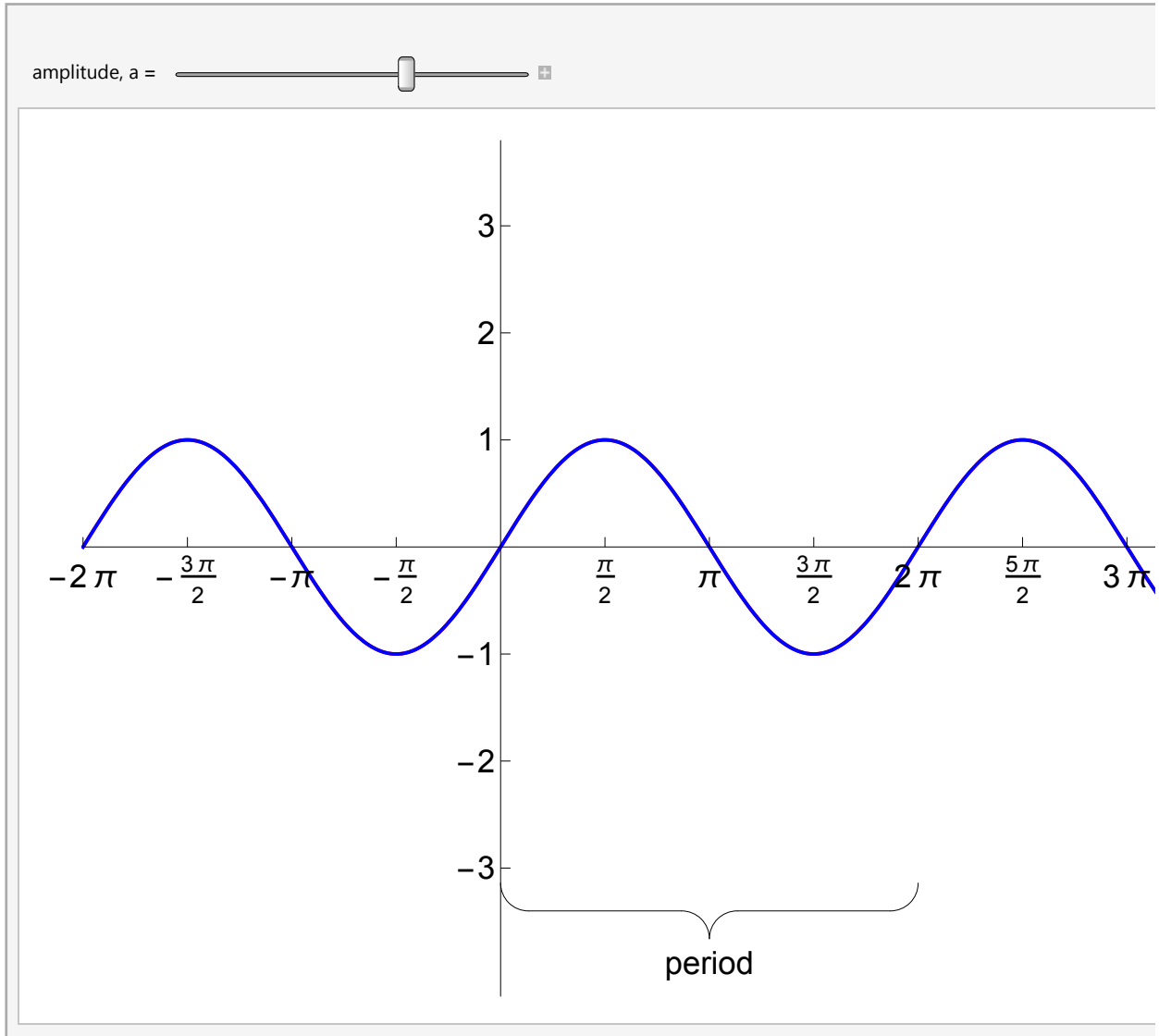
amplitude, a =


```
ManToGif[man, "amplitude", 2]
```

amplitude.gif

```
Manipulate[
 Show[
  Plot[{Sin[x], Sin[x] + b}, {x, -2 π, 4 π},
   Ticks →
    {{-2 π, -(3 π)/2, -π, -π/2, π/2, π, (3 π)/2, 2 π, (5 π)/2, 3 π, (7 π)/2, 4 π}, {-3, -2, -1, 1, 2, 3}},
   PlotRange → {{-2 π, 4 π + 2}, {-4.2, 3.8}},
   PlotStyle → {Directive[Red, AbsoluteThickness[2]],
     Directive[Blue, AbsoluteThickness[2]]},
   PlotLegends → LineLegend[{"f(x)      = sin(x)",
      "f(x) + b = sin(x) + b\n            = sin(x) + " <> If[b < 0, "(", ""] <>
       ToString[b] <> If[b < 0, ")", ""]}, LegendFunction → "Frame",
     LegendMargins → 6, LabelStyle → Directive[FontFamily → "Courier"]],
   BaseStyle → {FontSize → fontsize}],
  Graphics[braceLabel[{{4 π + 0.4, 1 + b}, {4 π + 0.4, b}}, "amplitude", 0.02]],
  Graphics[
   braceLabel[{{3 π, -3 - 0.4}, {0, -3 - 0.4}}, Rotate["period", 180 Degree], 0.02]],
  Graphics[{Dashed, Gray, Line[{{-2 π, b}, {4 π, b}}]}], ImageSize → 800
 ]
 , {{b, 0, "vertical shift, b = "}, -2, 2},
 BaseStyle → {FontSize → fontsize}, Initialization ⧴ (fontsize = 18;
   (* http://
    mathematica.stackexchange.com/questions/18012/label-area-on-plot *)
   braceLabel[{p1_, p2_}, lbl_, scale_: .02] :=
    {Arrowheads[{{scale, 0, {Graphics@Circle[{1, -1}, 1, {Pi / 2, Pi}], -1}}, {scale,
        1, {Graphics[{Circle[{-1, 1}, 1, {-Pi / 2, 0}], Inset[lbl, {0, 2}]}], 1}}}],
     Arrow[{p1, (p1 + p2) / 2}], Arrowheads[
      {{scale, 0, {Graphics@Circle[{1, 1}, 1, {Pi, 3 Pi / 2}], -1}},
       {scale, 1, {Graphics@Circle[{-1, -1}, 1, {0, Pi / 2}], 1}}}],
     Arrow[{(p1 + p2) / 2, p2}]};)]
```

vertical shift, b =
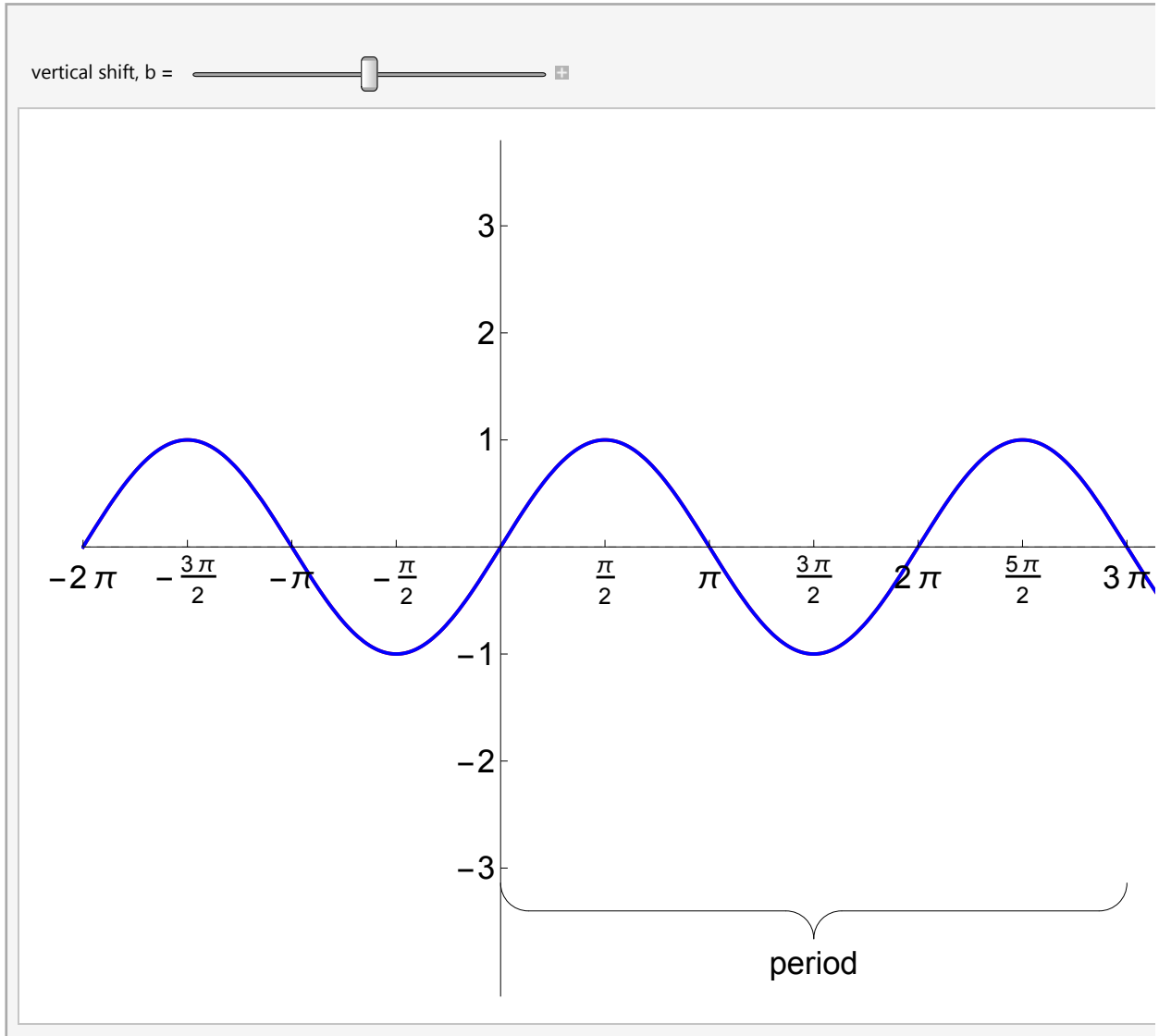
```
Manipulate[
 Show[
  Plot[{Sin[x], Sin[x - c]}, {x, -2 π, 4 π},
   Ticks →
    {{-2 π, - 3 π/2, -π, - π/2, π/2, π, 3 π/2, 2 π, 5 π/2, 3 π, 7 π/2, 4 π}, {-3, -2, -1, 1, 2, 3}},
   PlotRange → {{-2 π, 4 π + 2}, {-4.2, 3.8}},
   PlotStyle → {Directive[Red, AbsoluteThickness[2]],
    Directive[Blue, AbsoluteThickness[2]]},
   PlotLegends → LineLegend[{"f(x)      = sin(x)",
     "f(x - c) = sin(x - c)\n              = sin(x - " <> If[c < 0, "(", ""] <> ToString[
      c, TraditionalForm] <> If[c < 0, ")", ""] <> ")"}, LegendFunction → "Frame",
    LegendMargins → 6, LabelStyle → Directive[FontFamily → "Courier"]],
   BaseStyle → {FontSize → fontsize}],
  Graphics[braceLabel[{{4 π + 0.4, 1}, {4 π + 0.4, 0}}, "amplitude", 0.02]],
  Graphics[braceLabel[{{2 π + c, -3 - 0.4}, {c, -3 - 0.4}},
   Rotate["period", 180 Degree], 0.02]], ImageSize → 800
 ]
 , {{c, 0, "horizontal shift, c = "}, -2 π, π},
 BaseStyle → {FontSize → fontsize}, Initialization ⧴ (fontsize = 18;
  (* http://
   mathematica.stackexchange.com/questions/18012/label-area-on-plot *)
  braceLabel[{p1_, p2_}, lbl_, scale_: .02] :=
   {Arrowheads[{{scale, 0, {Graphics@Circle[{1, -1}, 1, {Pi / 2, Pi}], -1}}, {scale,
      1, {Graphics[{Circle[{-1, 1}, 1, {-Pi / 2, 0}], Inset[lbl, {0, 2}]}], 1}}}],
    Arrow[{p1, (p1 + p2) / 2}], Arrowheads[
     {{scale, 0, {Graphics@Circle[{1, 1}, 1, {Pi, 3 Pi / 2}], -1}},
      {scale, 1, {Graphics@Circle[{-1, -1}, 1, {0, Pi / 2}], 1}}}],
    Arrow[{(p1 + p2) / 2, p2}]};)]
```
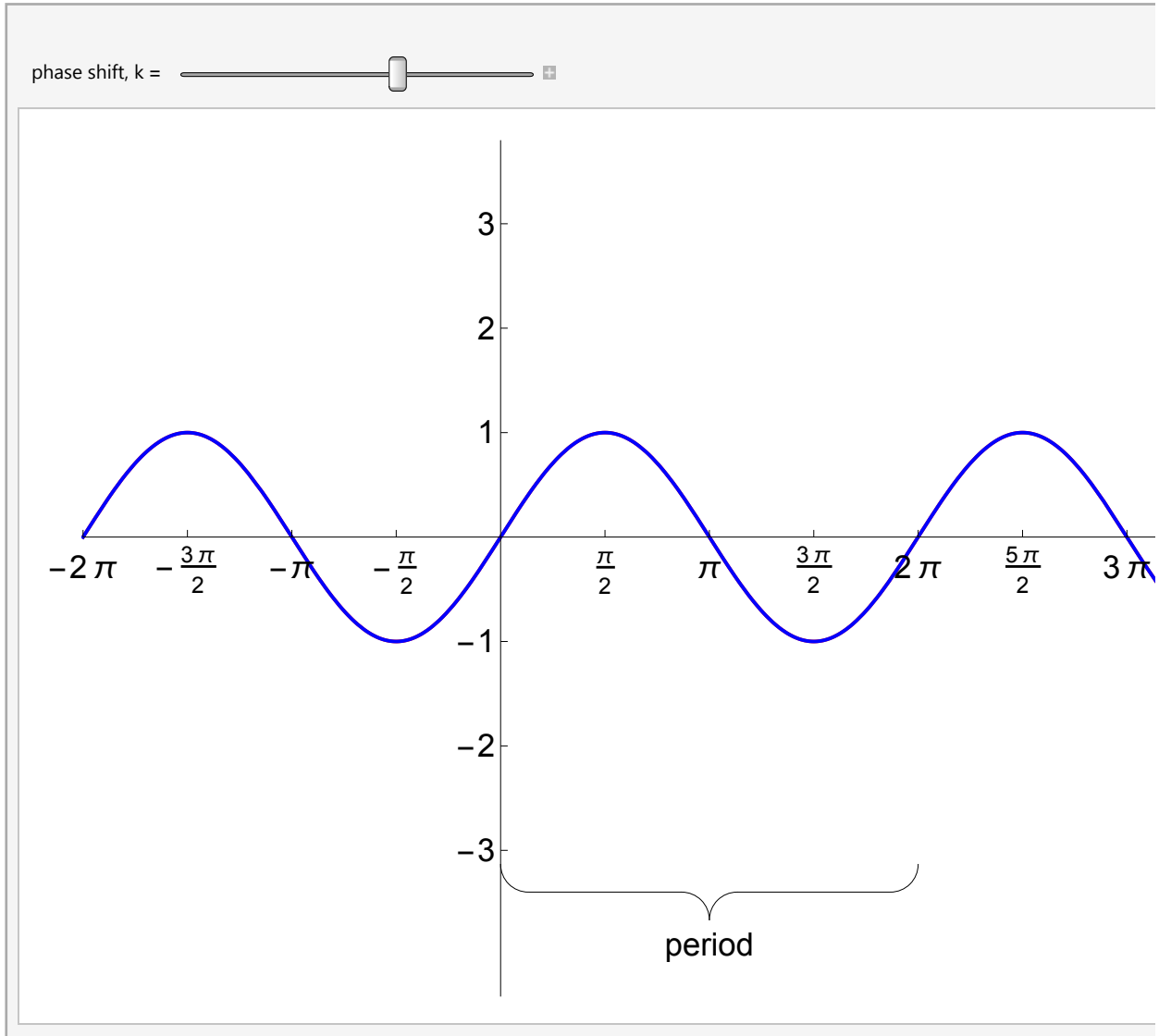
horizontal shift, c =
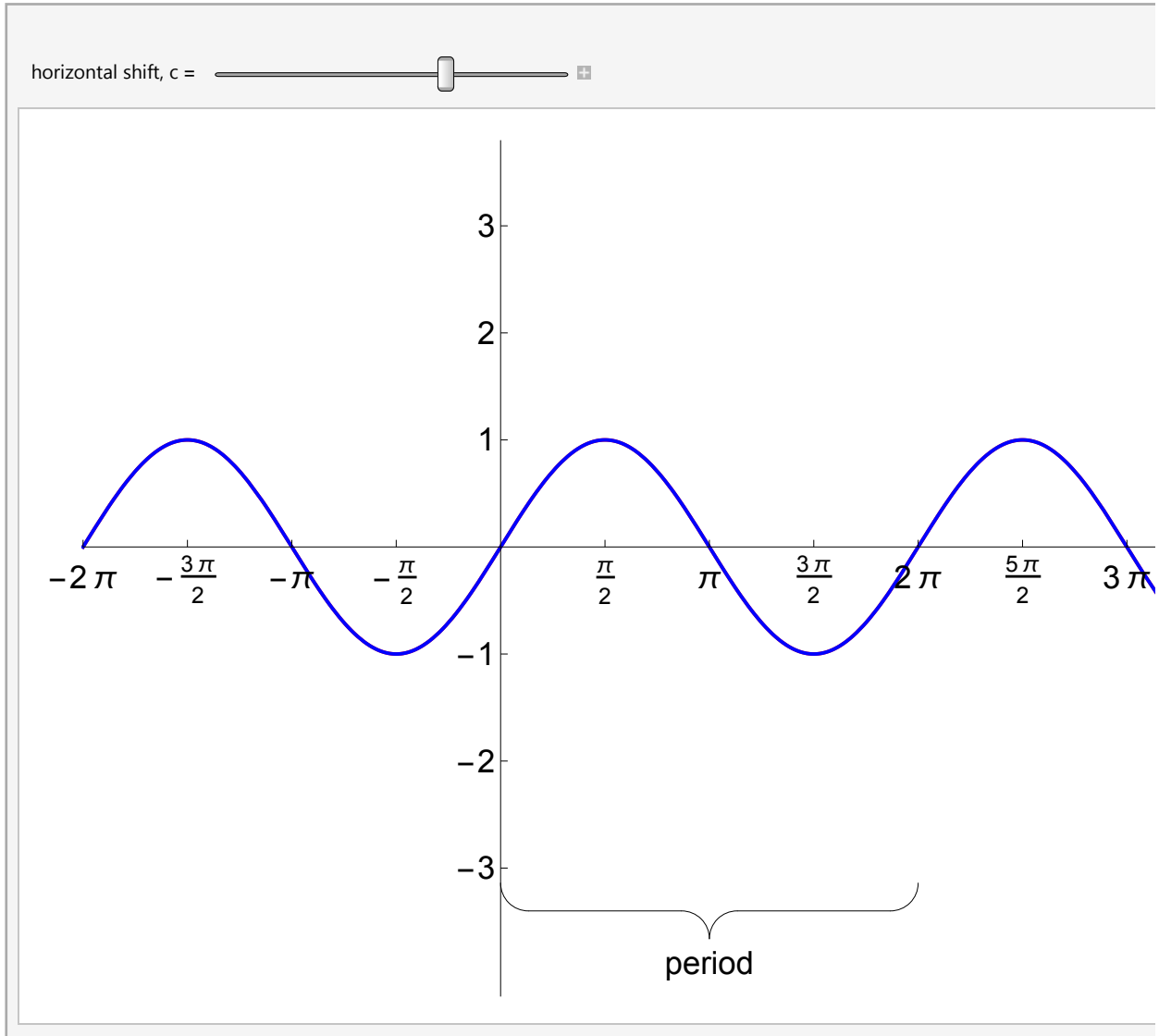
```
Manipulate[
 Show[
  Plot[{Sin[x], Sin[k x]}, {x, -2 π, 4 π},
   Ticks →
     {{-2 π, -(3 π)/2, -π, -π/2, π/2, π, (3 π)/2, 2 π, (5 π)/2, 3 π, (7 π)/2, 4 π}, {-3, -2, -1, 1, 2, 3}},
   PlotRange → {{-2 π, 4 π + 2}, {-4.4, 3.8}},
   PlotStyle → {Directive[Red, AbsoluteThickness[2]],
     Directive[Blue, AbsoluteThickness[2]]},
   PlotLegends → LineLegend[{"f(x)    = sin(x)",
       "f(k x) = sin(k x)\n          = sin(" <> ToString[k, TraditionalForm] <> " x)"},
     LegendFunction → "Frame", LegendMargins → 6,
     LabelStyle → Directive[FontFamily → "Courier"]],
   BaseStyle → {FontSize → fontsize}],
  If[k ≠ 0, Graphics[braceLabel[{{4 π + 0.4, 1}, {4 π + 0.4, 0}}, "amplitude", 0.02]],
   {}],
  If[k ≠ 0, Graphics[braceLabel[{{2 π / Abs[k], -3 - 0.4}, {0, -3 - 0.4}},
     Rotate["period", 180 Degree], 0.02]], {}], ImageSize → 800
 ]
 , {{k, 1, "phase shift, k = "}, -4, 4, 1/2},
 BaseStyle → {FontSize → fontsize}, Initialization ⧴ (fontsize = 18;
   (* http://
    mathematica.stackexchange.com/questions/18012/label-area-on-plot *)
   braceLabel[{p1_, p2_}, lbl_, scale_: .02] :=
    {Arrowheads[{{scale, 0, {Graphics@Circle[{1, -1}, 1, {Pi / 2, Pi}], -1}}, {scale,
        1, {Graphics[{Circle[{-1, 1}, 1, {-Pi / 2, 0}], Inset[lbl, {0, 2}]}], 1}}}],
     Arrow[{p1, (p1 + p2) / 2}], Arrowheads[
      {{scale, 0, {Graphics@Circle[{1, 1}, 1, {Pi, 3 Pi / 2}], -1}},
       {scale, 1, {Graphics@Circle[{-1, -1}, 1, {0, Pi / 2}], 1}}}],
     Arrow[{(p1 + p2) / 2, p2}]};)]
```

phase shift, k =

```mathematica
Manipulate[
 Show[
  Plot[{Sin[x], b + a Sin[k (x - c)]}, {x, -2 π, 5 π},
   Ticks → {{-2 π, -(3 π)/2, -π, -π/2, π/2, π, (3 π)/2, 2 π, (5 π)/2, 3 π, (7 π)/2, 4 π, (9 π)/2, 5 π},
     {-5, -4, -3, -2, -1, 1, 2, 3, 4, 5}},
   PlotRange → {{-2 π, 5 π + 2}, {-6.5, 5}},
   PlotStyle → {Directive[Red, AbsoluteThickness[2]],
     Directive[Blue, AbsoluteThickness[2]]},
   PlotLegends → LineLegend[{"f(x)                  = sin(x)",
      "b + a f(k (x - c)) = b + a sin(k (x - c))\n                      = " <>
       ToString[b] <> " + " <> If[a < 0, "(", ""] <> ToString[a] <> If[a < 0, ")", ""] <>
       " sin(" <> ToString[k, TraditionalForm] <> " (x - " <> If[c < 0, "(", ""] <>
       ToString[c, TraditionalForm] <> If[c < 0, ")", ""] <> "))"},
     LegendFunction → "Frame", LegendMargins → 6,
     LabelStyle → Directive[FontFamily → "Courier"]],
   BaseStyle → {FontSize → fontsize}],
  If[a ≠ 0 ⋀ k ≠ 0, Graphics[braceLabel[{{5 π + 0.4, Abs[a] + b}, {5 π + 0.4 , b}},
     "amplitude", If[Abs[a] < 1.5, 0.02/1.5 Abs[a], 0.02]]]], {}],
  Graphics[{Dashed, Gray, Line[{{-2 π, b}, {5 π, b}}]}],
  If[k ≠ 0, Graphics[braceLabel[
     {{2 π / Abs[k] + c , -5 - 0.4}, {c, -5 - 0.4}}, Rotate["period", 180 Degree],
     If[Abs[2 π / k] < 1.8, 0.02/1.8 Abs[2 π / k], 0.02]]], {}], ImageSize → 800
  ]
 , {{a, 1, "amplitude, a = "}, -3, 3}, {{b, 0, "vertical shift, b = "}, -2, 2},
 {{c, 0, "horizontal shift, c = "}, -2 π, π},
 {{k, 1, "phase shift, k = "}, -4, 4, 1 / 2},
 BaseStyle → {FontSize → fontsize}, Initialization ⧴ (fontsize = 18;
   (* http://
    mathematica.stackexchange.com/questions/18012/label-area-on-plot *)
   braceLabel[{p1_, p2_}, lbl_, scale_: .02] :=
   {Arrowheads[{{scale, 0, {Graphics@Circle[{1, -1}, 1, {Pi / 2, Pi}], -1}}, {scale,
        1, {Graphics[{Circle[{-1, 1}, 1, {-Pi / 2, 0}], Inset[lbl, {0, 2}]}], 1}}}],
     Arrow[{p1, (p1 + p2) / 2}], Arrowheads[
      {{scale, 0, {Graphics@Circle[{1, 1}, 1, {Pi, 3 Pi / 2}], -1}},
       {scale, 1, {Graphics@Circle[{-1, -1}, 1, {0, Pi / 2}], 1}}}],
     Arrow[{(p1 + p2) / 2, p2}]};)]
```

amplitude, a =

vertical shift, b =

horizontal shift, c =

phase shift, k =